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Fault detection for quality control of household appliances by non-invasive laser Doppler technique and likelihood classifier

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Abstract

This paper addresses the problem of developing an on-line diagnostic system for mechanical quality control of household appliances. The selection of a suitable measurement technique for feature extraction is discussed; the choice of a laser Doppler vibrometer technique and a laboratory measurement station for washing machines is presented. Vibration velocity and displacement are measured over a grid of points on the machine surface and data are stored in a database suitable for processing, both with good appliances and with defect ones with known defects. Features from the vibration velocity spectrum are used as the input to a likelihood classifier, which is shown to achieve very good classification scores. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Quality control of finished products is an essential part of the manufacturing process. Even though manufacturing has benefited from recent advances in automation and robotics, and thus has been able to reach a point of complete automation, quality control largely still relies on human operators and expertise. This is partly due to the nature of the problem, which is quite complex and requires the combination of advanced sensor technology and highly sophisticated artificial intelligence techniques. Recently, however, thanks to measurement system developments, and

while artificial intelligence is maturing, sophisticated automatic quality control systems are becoming part of standard production lines. An area where such ideas have been applied is acoustic quality control. For example, in ceramic tile production, the sound of a hammer hitting a tile, is used by a human to decide on its quality. Meier et al. [1] have used fuzzy data analysis methods to investigate the possibility of developing an automatic quality control system for ceramic tiles. Lukovich et al. [2] employed neural network classifiers for micro-mechanical equipment diagnostics and micro-mechanical product quality inspection. The use of acoustic signals is also used here, in addition to various other texture features. Fogliardi [3], used a Fuzzy C-Means algorithm for diagnostics of inductive motors characterised by undesired noise due to rotor shaft vibrations. This

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work is quite similar to ours but uses an acoustic signal instead of a vibration one and is therefore subject to disturbances in an industrial environment. Li and Wu [4] developed an on-line bearing monitoring system based on pattern recognition of bearing vibration signals.

Household appliances represent an interesting test case for diagnostic systems aimed at on-line quality control. In fact, although such products are characterised by a relatively low technological level and consequently a relatively low cost, the large number of pieces produced (in the range of 4000–5000/day in a medium-size plant), the dimension of the market and the competition among different manufacturers together with the increasing quality demand from the customers, all together fully justify investments in systems for on-line quality control.

Household appliances have many electromechanical subsystems which may have faults or may be improperly assembled. It should be emphasized that the products have several moving parts and therefore mechanical defects strongly affect overall performance, both in terms of functionality and in terms of vibro-acoustic comfort. The present state of the art, in terms of on-line industrial quality control on the fully assembled appliance, is generally limited to tests of the electrical parts of the machine; no mechanical diagnosis is at present carried out, although a large variety of mechanical defects are present in the final product.

In this paper we present such a system, which uses a measurement subsystem equipped with a laser Doppler vibrometer to produce extremely accurate vibration measurements, coupled to a data analysis engine which extracts appropriate features and classifies them into predefined fault classes. The whole project is a collaboration of six research partners within a European founded project (MEDEA) and aims at the development of a non-contact diagnostic system for mechanical testing of household appliances; the developed system is intended for on-line operation and will be capable of testing 100% of the production.

It is important to note that although what was developed is intended to be used for household appliances, most of the ideas underlying this project can be applied to other important industrial sectors,

such as the automotive industry, as well as to the general field of pattern recognition.

2. Instrumentation and methods

2.1. The measurement technique requirements

Mechanical diagnostics is traditionally performed by vibration and acoustic measurements [5–7]. The proposed system is aimed at the industry of household appliances, in particular washing machines (WMs) and dishwashers (DWs). The measurement technique employed for data extraction will have to satisfy several requirements and specifications and need to cope with the production system in terms of environment and logistics. In large industrial plants working at full capacity, only about 2 min will be available for testing each machine. Each machine will approach the test station on a pallet, then it will stop and be prepared for functionality tests. Testing time has to be divided into three main phases: machine set-up and connection to the power supply, actuators and sensing system; machine operation during test according to a prescribed sequence; and, finally, stopping of the machine and disassembling from the test station.

The production factory environment is very hostile and is characterised by extremely high levels of acoustic noise. Vibration may also be high. The humidity and temperature ranges are normal, but electromagnetic disturbances, both radiated and conducted, may be high, due to the many electrical and electronic devices operating on the production line.

Household appliances may exhibit several mechanical defects. It is necessary to measure vibrations at various locations on the machine casing in order to detect them. Therefore, the measurement system must be designed either as an array of sensors or a scanning system.

For an on-line diagnostic system operating in a structured environment, whose main degrees of freedom are fixed by the necessity to convey the machine inside and outside of the test station and putting it into operation, it is relevant to have the availability of a sensing system which can be automatically positioned at the chosen measurement

locations. The last, but not least, fact to be considered is that the household machine under test is a finished product which has functional and aesthetic characteristics which need to be preserved; therefore, non-invasive vibration sensing is necessary in order to avoid any surface damage.

There are several available measurement techniques for vibration sensing; in general, they measure displacement, velocity or acceleration and eventually deformation. The choice among them needs to take into account the aforementioned requirements.

Strain-gauges measure local deformation, but their application to industrial on-line diagnostics is not feasible. In fact, their installation is time consuming, no automatic positioning is possible, and sensors are invasive. Accelerometers, although widely used for machinery diagnostics, in this case have several limitations; it is in fact necessary to install them on the machine and this procedure is difficult to automate. Furthermore, they are invasive, causing local mass loading which is not negligible for the thin metal foils which form the household machine structure. Electromagnetic proximity sensors appear

as a solution to many of the problems related to intrusiveness, automatic positioning and the time needed to measure. However, this type of sensor is very sensitive to the characteristics of the target material, in the present case the metal foil with the paint layer on it. This would imply the necessity of frequent calibration of the sensor. Furthermore, most electromagnetic proximity sensors are strongly non-linear, have a relatively low measurement range and must be operated at a fixed stand-off distance from the target, all facts which impose the need for an accurate positioning of the sensor with respect to the target. In an automatic test station, where machines move in and out at a large rate, accurate sensor positioning should not be necessary. A further limitation of these sensors is their sensitivity to electromagnetic disturbances, which are relevant in the industrial environment. Therefore, optical proximity sensors remain the best candidate for vibration measurement in such conditions.

As part of the MEDEA project a special purpose interferometer will be developed, whose cost will be one order of magnitude lower than general-purpose laboratory vibrometers. The idea is to employ inte-

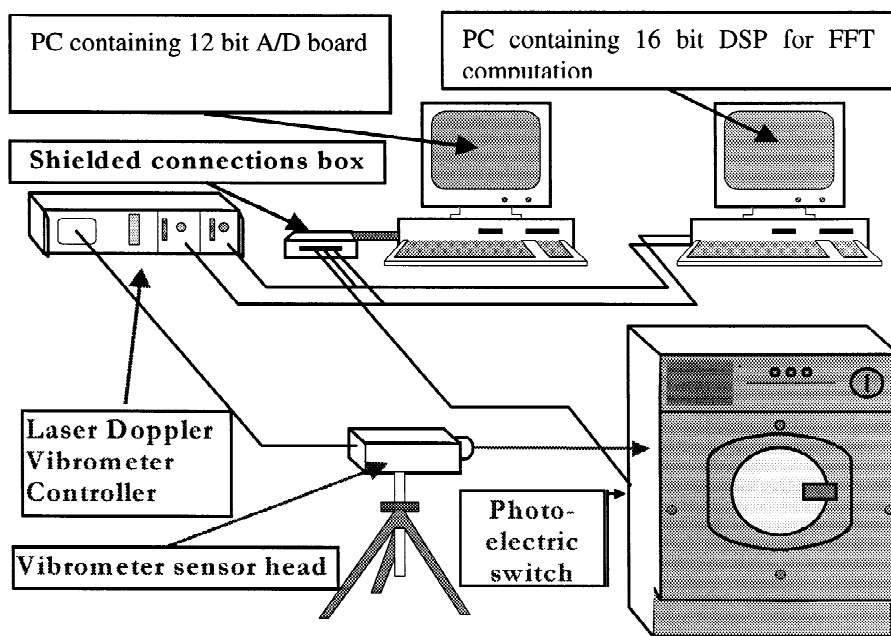


Fig. 1. The measurement system.

grated optics interferometers, operating in parallel through a fiber link in order to access simultaneously a grid of points. As a result the total cost of the diagnostic system will be kept to a minimum. For the preliminary laboratory tests, however, a single point laser Doppler vibrometer is employed.

2.2. The measurement procedure

The measuring system employed in these tests is described in Fig. 1.

The laser Doppler vibrometer is positioned around the washing machine and it measures two quantities, the velocity and displacement of vibration.

The laser Doppler vibrometer is a Mach-Zender interferometer; this instrument measures vibration velocity by demodulation of a frequency modulated signal produced by heterodyning the measurement beam with a reference beam, with a 40 MHz frequency shift. The measurement beam is frequency shifted by the Doppler effect caused by the vibrating object [10].

The measurement beam can be easily displaced from one measurement point to another; there exist versions of this instrument which displace the beam by moving mirrors. Therefore, on-line automatic testing is possible and fast. The system can operate from a large distance, which does not need to be kept constant. All electronic components can be isolated from interfering or modifying disturbances. Vibrations of the environment may be a concern, because the instrument measures with reference to its own position; vibration isolation is possible via conventional optical tables. Sensitivity, resolution, measure-

ment range and bandwidth make it an ideal instrument for contactless vibration measurement.

Measurements from the laser Doppler vibrometer are acquired by two PCs; the first one records the time signals sampled at 2000 Hz by a 12-bit A/D acquisition board, the second one is equipped with a 16-bit DSP board to produce real time spectra. Data are stored in a database ready for processing. In our application, we have to measure displacement and velocity from a grid of strategic points on the casing of the washing machine in order to produce the data necessary for the diagnostic system to classify the machines in classes (no defect, defect A, defect B, etc.). A TTL signal produced by a proximity sensor placed in front of the pulley, is also measured. It is used to calculate the drum's rotational velocity.

Measurements are performed on a selected grid of points; they are chosen on the basis of results of a first general study of the vibration of washing machines. An example of measurement points is shown in Fig. 2.

As part of this work, an optimum number of points is to be found. This is to be decided on the basis of cost per sensor and classification performance.

Measurements are made according to the following procedure: the machine is started and the drum accelerates from 0 to its maximum angular velocity, then a steady state is maintained for about 16 s, and finally the machine decelerates down to zero. This is done in order to excite vibrations of the machine at all typical frequencies of operation. The non-contact nature of the measurements allows the experimenter to neglect any mass loading effects. Beam position-

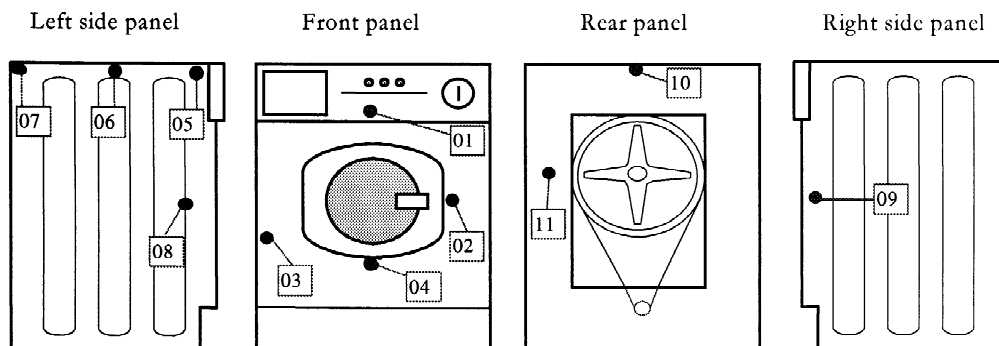


Fig. 2. A possible set of measurement points on the washing machine.

Table 1
Common defects present in a washing machine (from a survey at the Domotecnica Appliances Fair, Cologne)

Defect class definitions
Z: No defect
B: Electric motor clamping screws (released belt)
H: Releasing the shock absorber
M: Use of different types of spring
P: Pulley (distorted)

ing is rapid so that large amounts of data can be collected. In any case, the final version of the measurement system will employ multiple sensors operating simultaneously.

The defects introduced are reported in Table 1.

The defects introduced are the most common according to a survey carried out at one of the major European fairs, amongst all leading manufacturers. Firstly the no-defect measurements were performed. Then the defects were manually introduced on the WMs and the procedure was repeated.

For each measurement point (11 points on each WM) and for each WM (5 WMs were measured) 14 measurements were performed for each defect class (five classes). A total of 3850 files have been collected in the database.

2.3. Mathematical statement of the problem

As stated earlier, velocity and displacement of the WM's surface vibration are measured along with the WM's drum rotation speed. Initial experiments showed an erratic behaviour of the displacement data and it was decided to use only the velocity measurements. Thus, our problem can be stated as follows.

For a rotating machine we are given a set of N time series representing vibration velocity measurements (mm s^{-1}) from k measurement sensors,

$$\begin{aligned}
 X_o(i, j) &= \begin{bmatrix} x_o(1) \\ x_o(2) \\ \dots \\ x_o(k) \end{bmatrix} \\
 &= \begin{bmatrix} x_o(1, 1) & x_o(1, 2) & \dots & x_o(1, N) \\ x_o(2, 1) & x_o(2, 2) & \dots & x_o(2, N) \\ \dots & \dots & \dots & \dots \\ x_o(k, 1) & x_o(k, 2) & \dots & x_o(k, N) \end{bmatrix}
 \end{aligned}$$

and the rotation speed measurement (rpm),

$$v_o(j); \quad j = 1, \dots, N$$

where the subscript 'o' stands for 'original'. The machines belong to l distinct classes, $C_j; j = 1, \dots, l$, depending on the kind of defect present. For the situation at hand, $N \cong 40\,000, k = 4, l = 5$. A typical time series is shown in Fig. 3; from the time signals it is practically impossible to distinguish a defect machine [Fig. 3(B)] from a non-defect one [Fig. 3(A)].

The problem can thus be stated as follows [9]: "Given an initial training set, find a classifier that could adequately classify machines measured on-line in the production line." The meaning of 'adequately' is subjective but an acceptable figure should be over 95% of correctly classified machines. Furthermore, in the present situation an additional objective is present, because of low cost requirements: find the minimum number of sensors that fulfil the classification requirements.

In such classification problems, one is usually faced with two questions:

- What features to use as input to the classifier.
- Which classifier to use.

These questions can scarcely be answered easily. The usual approach is to try different combinations of features/classifiers and choose the best pair.

It is widely accepted that a good start for feature extraction is the transformation of the time signal into its frequency domain response. In this way, it is hoped to reduce the amount of information present in the time signal to a few meaningful harmonics or some other characteristic of the frequency plot. This is usually accomplished by the Fourier transform. However, before the FFT can be carried out, the part of the vibration signal that corresponds to steady-state (stable, stationary) operation of the washing machine, as implied by the rotation speed, must be extracted. This is necessary since the Fourier transform is valid only for stationary series (the transient part, however, can also be utilised using wavelet transforms). We accomplish this using the following algorithm (Pouliezos and Stavrakakis, 1994 [8]).

It is assumed that $v_o(j)$ is a Gaussian sequence. For a given window length n , the variable

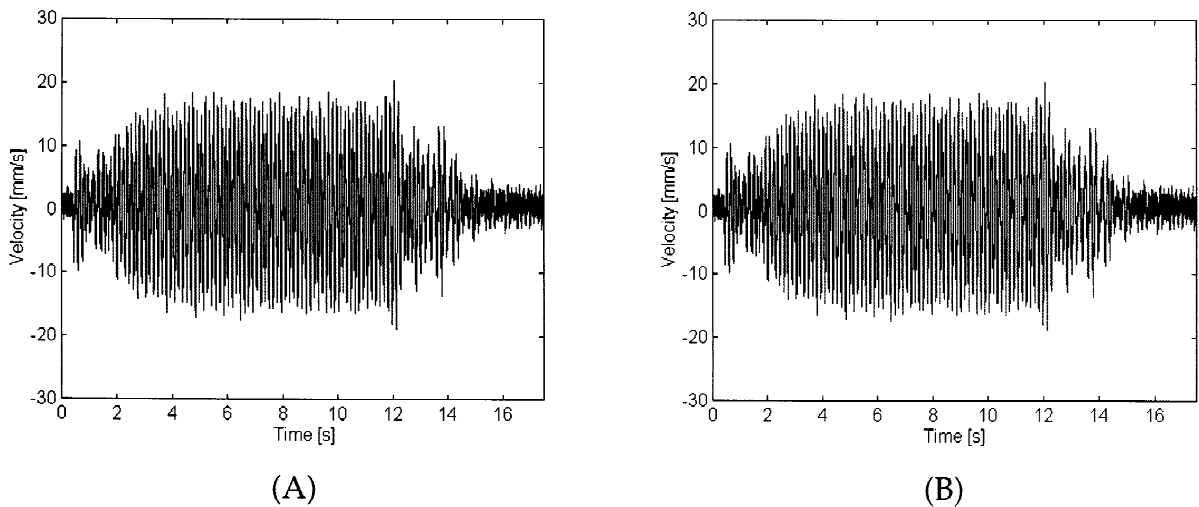


Fig. 3. Vibration velocity signals recorded from point 6 on: (A) non-defect washing machine; and (B) defect (defect B) washing machine.

$$w = \frac{1}{2} \frac{\sum_{i=1}^{n-1} (v_o(i+1) - v_o(i))^2}{\sum_{i=1}^n (v_o(i) - \hat{v}_o)^2} \tag{1}$$

where $\hat{v}_o = (1/n) \sum_{i=1}^n v_o(i)$ is the sample window mean, and has a probable value of 1. If n is bigger than 25, the variable

$$u = (1 - w)\sqrt{(n - 1)(n + 1)/2} \tag{2}$$

follows a zero-mean standardised Gaussian law. Using standard hypothesis testing, the following decision rule is used:

$u > \varepsilon$: dynamic state;

$u < \varepsilon$: steady state.

The threshold ε can be chosen considering standard P_d (probability of correct detection) and P_f (probability of false alarm) values or by simulation.

There is, however, one drawback in this algorithm: it may be too slow for on-line operation in the particular situation considered. Two ideas are used to overcome this problem:

- The original series $v_o(j)$ is undersampled so as to produce a shorter series. This has no effect on the separation process, since the original series is

sampled at high frequencies. The value that is used is 30, so that the new series has a length of about 1300.

- The relevant calculations are done iteratively from window to window since most terms in Eq. (1), (in fact all but two) are the same [8]. This greatly reduces the computational burden.

Thus, implementation of this procedure gives three break-points on the original $v_o(j)$ series,

- b_1 : start point for rising transient;
- b_2 : start point of steady state;
- b_3 : end point of steady state.

thus producing two series:

- $v_t(j) = \{v_o(j); j = b_1, \dots, b_2 - 1\}$ (transient part);
- $v_s(j) = \{v_o(j); j = b_2, \dots, b_3\}$ (steady-state part).

Results of this procedure are shown in Fig. 4, with a window length of 40. It shows machine rotational velocity during a test, the steady-state index and vibration velocity during steady state. The first part of the figure shows the rotational velocity against sample number for a machine codenamed ‘O1az2c08’. The second part of the figure shows the sample history of the test statistic u used to separate

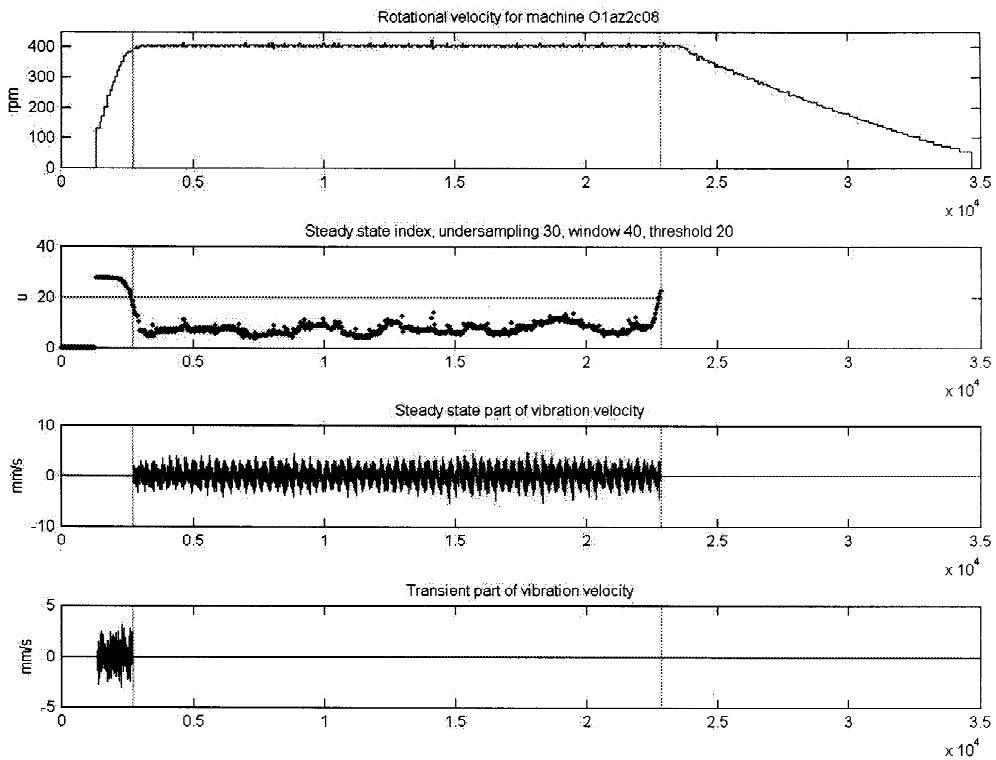


Fig. 4. Rotational velocity of the washing machine drum during the test procedure (upper figure); steady-state index u (middle figure); measured vibration velocity in the steady state (lower figure).

the transient from the steady-state part of the rotational velocity series, as defined in Eq. (2). Also shown are the threshold ($\cdot \cdot \cdot$, value = 20), used for deciding whether the rotational velocity series is in the dynamic or static condition, and the two break points (indicated by the vertical lines on all graphs), b_2 and b_3 , which specify the start and finish of the steady-state part. The third and fourth parts of the figure show the series $v_s(j)$ and $v_t(j)$, i.e. the corresponding steady state and transient part of the vibration velocity time series.

As can be seen the results are not quite as expected, since u does not seem to be zero mean. One possible explanation may be that the assumption of being Gaussian is not valid. This problem was overcome by raising the threshold to $\varepsilon = 20$.

Now using the transient and steady-state parts of the rotational velocity series, the transient and steady-state parts of the vibration velocity series (for each measurement point i) are extracted,

$$\begin{aligned}
 X_t(i, j) &= \begin{bmatrix} x_t(1) \\ x_t(2) \\ \dots \\ x_t(k) \end{bmatrix} \\
 &= \begin{bmatrix} x_t(1, 1) & x_t(1, 2) & \dots & x_t(1, N) \\ x_t(2, 1) & x_t(2, 2) & \dots & x_t(2, N) \\ \dots & \dots & \dots & \dots \\ x_t(k, 1) & x_t(k, 2) & \dots & x_t(k, N) \end{bmatrix} \quad (\text{transient part}) \\
 X_s(i, j) &= \begin{bmatrix} x_s(1) \\ x_s(2) \\ \dots \\ x_s(k) \end{bmatrix} \\
 &= \begin{bmatrix} x_s(1, 1) & x_s(1, 2) & \dots & x_s(1, N) \\ x_s(2, 1) & x_s(2, 2) & \dots & x_s(2, N) \\ \dots & \dots & \dots & \dots \\ x_s(k, 1) & x_s(k, 2) & \dots & x_s(k, N) \end{bmatrix} \quad (\text{steady-state part})
 \end{aligned}$$

Having broken the original time-series into its transient and steady-state parts, the question arises as

to what features to consider for each part, i.e. a feature vector,

$$f = \begin{bmatrix} f_t \\ f_s \end{bmatrix}$$

is sought, where f_t is the feature vector corresponding to the transient part and f_s to the steady-state part. The structures of f_t and f_s are as follows:

$$f_t = \begin{bmatrix} f_t(1) \\ f_t(2) \\ \dots \\ f_t(k) \end{bmatrix} = \begin{bmatrix} [f_t(1,1) \ f_t(1,2) \ \dots \ f_t(1,n_t)]^T \\ [f_t(2,1) \ f_t(2,2) \ \dots \ f_t(2,n_t)]^T \\ \dots \\ [f_t(k,1) \ f_t(k,2) \ \dots \ f_t(k,n_t)]^T \end{bmatrix}$$

$$f_s = \begin{bmatrix} f_s(1) \\ f_s(2) \\ \dots \\ f_s(k) \end{bmatrix} = \begin{bmatrix} [f_s(1,1) \ f_s(1,2) \ \dots \ f_s(1,n_s)]^T \\ [f_s(2,1) \ f_s(2,2) \ \dots \ f_s(2,n_s)]^T \\ \dots \\ [f_s(k,1) \ f_s(k,2) \ \dots \ f_s(k,n_s)]^T \end{bmatrix}$$

The sizes n_t , n_s of each feature sub-vector are to be determined, i.e. the total size of the feature vector is $kn_t + kn_s$.

Since $x_t(i)$ and $x_s(i)$ have different but known statistical properties, i.e. the first is non-stationary while the second stationary, the first step towards feature extraction is largely dictated by these properties. Therefore, $x_t(i)$ may be transformed using wavelet transforms while $x_s(i)$ may be transformed using Fourier transforms. Preliminary results of the steady-state part have produced extremely good results, so this approach will be now presented.

From the series $x_s(i)$ we thus obtain a normalised transformed series,

$$X_s^f(i) = 2|c(i)|/N_f; \quad i = 1, \dots, N_f$$

where, $c(i)$ is the (complex) coefficient of the Fourier series expansion of $f(i)$, i.e.

$$f(m) = \frac{1}{N_f} \sum_{k=0}^{N_f-1} c(k) e^{(j2\delta km)/N_f}; \quad m = 1, 2, \dots, N_f$$

and N_f is the length of the series. This form has the advantage that the amplitudes in the time series are preserved in the frequency spectrum, e.g. a unit sinusoid in the time domain has a unit coefficient in the frequency domain. Because of computational speed we choose N_f to be a power of 2:

$$N_f = 16384$$

The frequency range of interest is 0–200 Hz. It is

in this range, in fact, that the mechanical and structural frequencies in washing machines of interest are found. Having obtained the FFT transform $X_s^f(i)$ of $x_s(i)$ the next question refers to which characteristics could be used as features. In the relevant literature there exist numerous suggestions and it is a matter of examining the problem at hand, and trial and error techniques, in order to arrive at an appropriate set. One of the issues that must be faced is by which procedure one decides which set of features is ‘best’. An obvious answer would be: ‘the set which best classifies the machines’. Thus, the problem of feature selection is coupled to the problem of classifier selection. If one is to choose amongst n feature sets and m classifiers, then nm couples should be considered. Since our training set is rather small, we adopted the following verification procedure:

- for each feature set i and classifier j
- for each machine k
- train the classifier using $N-1$ machines and leaving machine k out
- present machine k to the classifier
- calculate machines correctly classified

The ratio $p = (\text{machines correctly classified}/N)$ then gives an indication of the relative merit of each (feature set)–classifier pair. The pair with the highest ratio is then selected as the optimum. Furthermore, since, as stated earlier, the number of sensors must be optimum, the above procedure was repeated for various combinations of measurement points, and the best combination selected for final adoption.

3. Results

Several tests were run to get a first idea as to which features could be significant. Experiments with a number of possibilities for FFT characteristics were performed, such as,

- spectrum peaks;
- frequencies where spectrum peaks occur;
- energy content of spectrum slices;
- peaks at odd harmonics . . .

as well as with a number of classifiers, such as,

- c-means;
- fuzzy c-means;
- distances (Euclidean, Mahalanobis, Bayes);
- likelihood;
- ANFIS . . .

Results showed that the best combination is:

- peaks at odd harmonics classified by likelihood methods.

Odd harmonics are odd multiples of the base harmonic which occurs at (steady-state rpm/60) ≈ 400/60 = 6.7. The likelihood classifier is a supervised learning method. Details are described in Appendix A.

The eight first odd harmonics were calculated and the peaks within a narrow band around them were found. The measurement points used were (1, 2, 6, 8, 9) as in these points there existed a full, compatible measurement set. The defect class set was,

$$C_j = \{Z, B, H, M, P\}$$

The total number of machines for the five points were 270. Prior probabilities for each class are calculated using the frequency formula (for the case where a machine type Z is left out for generalisation):

$$P_j = \left\{ \frac{65}{269} \quad \frac{65}{269} \quad \frac{60}{269} \quad \frac{40}{269} \quad \frac{40}{269} \right\} \\ = \{0.24 \quad 0.24 \quad 0.222 \quad 0.148 \quad 0.148\}$$

If only the detection of a defective (X) or a non-defective machine is required, the class list is

$$C_j = \{Z, X\}$$

where,

$$X = \{B, H, M, P\}$$

with prior probabilities (in the case where a type Z is left out)

$$P_j = \left\{ \frac{65}{269} \quad \frac{205}{270} \right\} = \{0.24 \quad 0.76\}$$

We experimented with different combinations of 2, 3 and 4 measurement points, with the results reported in Table 2 (only the best combination is shown).

Table 2
Reclassification results for different combinations of measurement point combination

Points	Correct	Percentage
<i>Two classes</i>		
1, 2	256	0.9481
1, 6, 9	261	0.9667
1, 2, 6, 9	261	0.9667
<i>Five classes</i>		
6, 8	260	0.9630
2, 8, 9	262	0.9704
1, 2, 6, 9	257	0.9519

The best combination (2, 8, 9) produced the results reported in Table 3 for each class.

4. Discussion and conclusion

A laboratory test station based on a laser Doppler vibrometer suited to fault detection and classification of washing machines, and utilising measurements of vibration velocity or displacement, has been developed.

The laser Doppler vibrometer plays a fundamental role in permitting the design of a test system capable of non-contact measurement on many points, in a hostile environment; the optical instrument allows also multipoint fast measurements, which is a requirement of the system for industrial application.

The data from the measurement subsystem is fed to the computer-based feature extraction and classification subsystem for subsequent diagnosis of the state of the washing machine under test. Preliminary

Table 3
Reclassification results for each class using the optimum measurement point combination

Class	Machines in sample	Correctly classified	Percentage
<i>Five classes</i>			
Z	65	63	0.9692
B	65	62	0.9538
H	60	59	0.9833
M	40	39	0.9750
P	40	39	0.9750
Total	270	262	0.9704

results, using odd harmonics peaks as features and a likelihood classifier, were very satisfactory. Data processing of signals coming from a strategical set of measuring points have permitted 97% success rates in fault detection. Percentages of 95%, in the worst case, and 97% in the best case, have been obtained in the identification of the class of the defect.

The procedure and the technique developed have been designed to be used in on-line operation. Results obtained are going to be used for the final prototype, which will be used on a washing machine production line.

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Appendix A

The feature vector finally adopted is

$$f_s = \begin{bmatrix} f_s(2) \\ f_s(8) \\ f_s(9) \end{bmatrix} = \begin{bmatrix} f_s(2, 1) \\ \dots \\ f_s(2, 8) \\ f_s(8, 1) \\ \dots \\ f_s(8, 8) \\ f_s(9, 1) \\ \dots \\ f_s(9, 8) \end{bmatrix}$$

where (2, 8, 9) refer to the optimum triple of measurement points. The classifier works as follows.

For each training set of $N - 1$ machines for class j

we calculate the mean vector (i.e. the class centre,

$$m_s^j = \frac{1}{N-1} \begin{bmatrix} \sum_{i=1}^{N-1} f_s^i(2, 1) \\ \dots \\ \sum_{i=1}^{N-1} f_s^i(2, 8) \\ \dots \\ \sum_{i=1}^{N-1} f_s^i(8, 1) \\ \dots \\ \sum_{i=1}^{N-1} f_s^i(8, 8) \\ \dots \\ \sum_{i=1}^{N-1} f_s^i(9, 1) \\ \dots \\ \sum_{i=1}^{N-1} f_s^i(9, 8) \end{bmatrix} = \frac{1}{N-1} \begin{bmatrix} \sum_{i=1}^{N-1} f_s^i(1) \\ \dots \\ \sum_{i=1}^{N-1} f_s^i(8) \\ \dots \\ \sum_{i=1}^{N-1} f_s^i(9) \\ \dots \\ \sum_{i=1}^{N-1} f_s^i(16) \\ \dots \\ \sum_{i=1}^{N-1} f_s^i(17) \\ \dots \\ \sum_{i=1}^{N-1} f_s^i(24) \end{bmatrix} = \begin{bmatrix} m_s^j(1) \\ m_s^j(2) \\ \dots \\ m_s^j(24) \end{bmatrix}$$

(where the superscript i denotes sample number and j class number) and covariance matrix (i.e. class dispersion)

$$C_s^j = \frac{1}{N-2} \begin{bmatrix} \sum_{i=1}^{N-1} (f_s^i(1) - m_s^j(1))^2 & \sum_{i=1}^{N-1} (f_s^i(1) - m_s^j(1))(f_s^i(2) - m_s^j(2)) & \dots & \sum_{i=1}^{N-1} (f_s^i(1) - m_s^j(1))(f_s^i(24) - m_s^j(24)) \\ \sum_{i=1}^{N-1} (f_s^i(1) - m_s^j(1))(f_s^i(2) - m_s^j(2)) & \sum_{i=1}^{N-1} (f_s^i(2) - m_s^j(2))^2 & \dots & \sum_{i=1}^{N-1} (f_s^i(2) - m_s^j(2))(f_s^i(24) - m_s^j(24)) \\ \dots & \dots & \dots & \dots \\ \sum_{i=1}^{N-1} (f_s^i(1) - m_s^j(1))(f_s^i(24) - m_s^j(24)) & \sum_{i=1}^{N-1} (f_s^i(24) - m_s^j(24))(f_s^i(2) - m_s^j(2)) & \dots & \sum_{i=1}^{N-1} (f_s^i(24) - m_s^j(24))^2 \end{bmatrix}$$

The prior probabilities for each class p_j , are also calculated from,

$$p_j = N_j / N - 1$$

where N_j is the total number of machines belonging to class j , used in the training set. The training set is composed of $N - 1$ machines, then for the remaining N th machine, the feature vector is calculated

$$f_s^N = \begin{bmatrix} f_s^N(1) \\ \vdots \\ f_s^N(24) \end{bmatrix}$$

Assuming a normal distribution of pattern vectors in the feature space, the probability that f_s^N belongs to class C_j is given by (Barschdorf, 1991 [9])

$$p(f_s^N | j) = \frac{1}{((2\delta)^n \det[C_s^j])^{1/2}} \exp[-0.5(f_s^N - m_s^j)^T \times [C_s^j]^{-1}(f_s^N - m_s^j)]$$

while the likelihood that f_s^N originated from class C_j is given by

$$\ell_j = \frac{[(f_s^N | j)p(j)]}{p(f_s^N)} \tag{A.1}$$

where $p(j)$ is the multivariable probability distribution of class j and $p(f_s^N)$ denotes the prior probability that the feature vector belongs to class j . In this way, the fact that fault modes are less likely to occur is taken explicitly into account. If features are uncorrelated and normally distributed, Eq. (A.1) is easier calculated, using

$$\ell_j = \ln[p(j)] - \frac{1}{2} \ln \det[C_s^j] - \frac{1}{2} (f_s^N - m_s^j)^T [C_s^j]^{-1} (f_s^N - m_s^j)$$

A fuzzy-like classifier can be obtained if the likelihoods are normalised

$$L_n(j) = \frac{1}{\sum_{l=1}^I \ell(j)} \begin{bmatrix} \ell(1) \\ \vdots \\ \ell(l) \end{bmatrix}$$

The j th element of this vector is the likelihood that the machine belongs to class j . Therefore, the decision is made that the machine belongs to the class that has the maximum likelihood

$$(\text{class index}) = \max_j (\ell(j))$$

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