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Optimal Robot Speed Trajectory by Minimization of the Actuator Motor Electromechanical Losses

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Abstract. This paper considers the control problem of a robotic manipulator with separately excited dc motor drives as actuators. An innovative method is proposed which achieves robot speed-control requirements, with simultaneous minimization of total electromechanical losses, while the drives follow the desired speed profiles of the robot joints under various loads and random load disturbances. If there is no demand for a specific speed profile, the optimal speed trajectory is determined by minimizing an electromechanical losses criterion. Controllable energy losses, such as armature copper losses, armature iron losses, field copper losses, stray load losses, brush load losses, friction and windage losses, can be expressed proportionally to the squares of the armature and the field (exciting) currents, the angular velocity and the magnetic field flux. The controllable energy loss term is also included in the optimal control integral quadratic performance index, defined for the whole operation period. Thus the appropriate control signals required for following the desired trajectory by simultaneous energy loss minimization for the whole operation interval are achieved. Two case studies of optimal robot control with and without minimization of actuator energy losses are presented and compared, showing the energy savings that can be achieved by the proposed methodology.

Key words: industrial robots, electric actuators, dc motor actuators, direct drive rotary actuators, robotic manipulators, electromechanical losses, loss minimization, optimal speed trajectory.

Nomenclature, type symbols, abbreviations and robot dc motor drive parameters

i, ω, etc.	small letters: instantaneous values
I, Ω, etc.	capital letters: normalized values
f	function
i_0, ω_0, etc.	zero subscript: nominal values
i_a	armature current
i_e	exciting current
u_a	armature voltage

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u_e	stator (field) voltage
R_α	rotor resistance
L_α	rotor inductance
R_e	field resistance
L_e	field inductance
φ_e	air-gap flux
Φ_{flux}	normalized air-gap flux
θ	angle rotation
ω	motor speed
m_{0L}	extrapolated stalled rotor torque ($= c\varphi_{0e}i_{0a}$)
m_L	applied load torque
m_d	driving (electric) torque ($= c\varphi_e i_a$)
c	motor constant
C_1	coefficient of eddy-current loss (see Table III of Appendix)
C_2	coefficient of hysteresis loss (see Table III of Appendix)
C_3	coefficient of stray loss
a, β	constants corresponding to the linear and non-linear parts of the magnetization curve
N_e	winding N turns
J	motor load inertia
k_L	viscosity coefficient
$\boldsymbol{R}$	2×2 positive definite weighting matrix
p, q	positive weighting factors
T_a	electrical time constant
T_{mn}	mechanical time constant
T_{0e}	stator time constant
T	displacement duration time
σ_F	coefficient of Eddy-current magnetic loss of iron laminations (see Table II of Appendix)
σ_H	coefficient of hysteresis magnetic loss of iron laminations (see Table II of Appendix)
$\boldsymbol{x}$	state vector ($= [I_\alpha(t)\Omega(t)\Phi_{\text{flux}}(t)]^{\mathrm{T}}$)
$\boldsymbol{u}$	control vector ($= [V_\alpha(t)V_e(t)]^{\mathrm{T}}$)
$\boldsymbol{x}_0, \boldsymbol{x}_T$	state vector at times $t = 0$ and $t = T$ (boundary conditions)
$\boldsymbol{u}_l, \boldsymbol{x}_l$	control and state lower bounds
$\boldsymbol{u}_u, \boldsymbol{x}_u$	control and state upper bounds

1. Introduction

Energy conservation through energy efficient practices, is a current requirement in many industries. The use of robots is ever increasing in industry, since robots are made cheaper, more efficient and more reliable. The energy efficient operation of any robot depends on the right specifications, design and selection at the project initiation/expansion stage. In the cases that this is not done or the robot's initial use is changed, energy can be saved by applying a method to control the 'quantities' which minimize the energy losses.

The robot can be actuated by different types of drives (hydraulic, gas, electric). The electric drives are the most common [25], including special types of dc and ac motors. Each robot drive has some particular advantage over the others under certain circumstances. Robot manufacturers are well aware of this and have developed products to meet specific needs and market demand. Direct current (dc) separately excited motors are very often used as actuators in industrial robots ([33, 34]). These actuators have low friction, small size, high speed, low construction cost, no gear backlash, operate safely without the use of limit switches and generate moderate torque at a high torque to weight ratio. Applications include camera pointing, robot hands, and robot legs. dc motors are preferred over ac motors because of their lower cost (which however tends to change in favour of ac motors) and ease of controller implementation, because their mathematical model is simpler. Even though the future trend could move towards ac motors, there exist a large number of robots in operation that could benefit from the proposed methodology. Furthermore this approach is not limited to the separately excited dc motors that are used in the simulations, but could easily be applied to any kind of motor (e.g. permanent-magnet, ac, induction) as long as their mathematical model is known.

Because the motor speed is largely sensitive to torque variations, the energy which is dissipated in a dc motor actuator strongly depends on its speed profile. In this case the full motor actuator excitation is not required ([7–9, 20, 21, 27]), and it may be adjusted according to the load requirements. However, partially loaded robots controlled with energy saving criteria can dissipate less energy than without. The energy saving can be achieved by minimization of the robot input power supply and/or by determining the optimal robot speed trajectory. The speed trajectory is determined if a reference speed profile is not required.

The basic ways of reducing losses in electric drives are incorporated through the principles of control. These can be divided into two major categories:

(a) Controllers that minimize losses during a certain motor's operation profile ([4, 18, 32]).
(b) Controllers that minimize losses during a steady state motor's operation profile ([7–9, 12, 16, 17, 21, 26]).

However, the previously mentioned issues (a), (b) have been addressed separately in the existing literature. Both enable the design of two kinds of controllers: "Loss-Model Controller" (LMC) ([4, 7–9, 12, 16–18, 21, 32]) and "Search Controller" [26].

The LMC measures the speed and armature current, satisfies the condition for minimum motor electromechanical losses and determines the optimal excitation value. Thus, the motor is maintained at its maximum efficiency and keeps a desired speed, by means of both armature voltage control and field voltage control. The main disadvantages of this controller are:

- It does not operate well when the load disturbances are big.
- It is rather complex in implementation. In order to overcome this difficulty certain types of important losses such as iron losses and stray losses, are usually omitted.
- It introduces a considerable delay and thus the response of the dc motor to abrupt load changes is not satisfactory.

The "Search Controller" does not satisfy the condition for minimum electromechanical losses, but has the ability to regulate speed without a specialised knowledge of the motor model, by direct measurement of the incoming power to the drive and by minimizing the input power by testing. This controller does not operate well when the load disturbances are small. The main disadvantages of this controller are:

- It is not possible to detect effectively when the limits of the motor efficiency to be improved are too narrow. This occurs when the motor operates under nominal power conditions and high efficiency.
- It is not possible to detect efficiently the new minimum of losses under variable motor loads.

This paper proposes an innovative method for the determination of the optimal excitation of a robot dc motor actuator with simultaneous minimisation of its electromechanical losses, by following a desired speed profile under a fluctuating load. The computation of the optimal motor excitation involves the complete knowledge of the motor design parameters and the motor losses model. This method is derived on the basis of an improved optimal regulator theory and is implemented easily by means of a computer program, which has been developed by solving the optimal non-linear control problem under minimal energy consumption and with respect to the robot operational constraints. The total controllable energy losses are precisely calculated and included in the optimal control quadratic performance index, defined for the whole operation period of the robot trajectory following. In practice, only the armature and the field circuit losses can be considered controllable ([7–9, 21]). Obviously, reduction of these losses by decreasing the armature current and magnetic flux to the necessary minimum levels results in reduction of the core losses and stray-load losses corresponding to the minimum levels. Moreover, the proposed control scheme guarantees the robot stability and is easy to implement, through a lookup table procedure.

The rest of the paper is organized as follows: Section 2 defines the motor losses and their minimisation condition. Section 3 describes the mathematical model of the robot motor and Section 4 formulates the relevant optimal control problem both when a fixed speed profile is required and when the speed profile is calculated as part of the optimisation procedure. Section 5 describes the numerical algorithm used for solving the optimisation problems and Section 6 discusses the simulation results. The paper concludes with Section 7.

2. Minimization of Losses in a Robot dc Motor Drive

A comprehensive discussion for modelling the electromechanical losses of dc drives is first presented by Kostenko and Piotrovsky [20].

The power terms involved in modelling the electromechanical losses of robot dc motor drives are [26]:

1. *The armature copper loss*, P_a, calculated as $i_a^2 R_a$. These losses are due to the electric current, i_a, flowing through the stator windings; R_a is the rotor resistance.
2. *The field copper loss*, P_e, calculated as $i_e^2 R_e$. These losses are due to the electric current, i_e, flowing through the rotor windings; R_e is the field resistance.
3. *The armature iron loss*, P_i, calculated as $C_1 \varphi_e^2 \omega^2 + C_2 \varphi_e^2 \omega^2$, where C_1 and C_2 are the coefficients of hysteresis loss and eddy current loss, respectively, ω is the motor speed and φ_e is the air-gap flux. The values of these coefficients depend on the motor material properties, the number of pole pairs and the rotor length ([20, 26]).
4. *The friction and windage loss*, P_m, is a function of the motor speed, $f_\ell(\omega)$.
5. *The stray load loss*, P_{st}, calculated as $C_3 \omega^2 i_a^2$, where C_3 is the stray losses coefficient. These losses arise in both the copper and iron parts of the motor.
6. *The brushes contact loss*, P_b, calculated as $2 v_b i_b$. These losses are due to the electric current, i_b, flowing through the dc motor drive brushes; v_b is the voltage drop across the brushes.

The mechanical losses are not controllable at a desired speed because these are essentially determined by the rotational speed of the motor system.

Thus, the power loss due to electromechanical losses of the robot dc drive can be written as follows:

$$P_{\text{loss}} = i_a^2 R_a + i_e^2 R_e + (C_1 \omega^2 + C_2 \omega) \varphi_e^2 + C_3 \omega^2 i_a^2 + 2 v_b i_a + f_\ell(\omega). \tag{1}$$

The relationship between i_a and the air-gap flux φ_e is obtained from the dc drive torque equation:

$$T_m = i_a \varphi_e. \tag{2}$$

The motor magnetization curve is described by the equation:

$$\varphi_e = \frac{\alpha i_e}{1 + \beta i_e}. \tag{3a}$$

Due to loss minimization, the motor must operate through a reduced excitation. The consequence is that the motor operates mainly at the linear part of its magnetization curve. Therefore, it is reasonable to assume a linear magnetization curve, i.e.:

$$\varphi_e = \alpha i_e. \tag{3b}$$

In order to derive the dc motor drive electromechanical losses minimization conditions, it is assumed that the motor is in steady state (both speed and torque are constant). The efficiency can be increased by reducing the iron loss, that is, the field current. Then, the armature current should be increased to obtain the required motor torque. Because the field copper loss and the armature iron loss decrease and the armature copper loss increases, the total losses vary with the combination of the armature current and the field current. If the field current and the armature current are controlled independently, many combinations of i_e and i_a will exist to meet the given operating point (ω, i_a, i_e, m_d), where m_d is the electric torque. The loss minimization condition is derived from the equation:

$$\left.\frac{\partial P_{\text{loss}}}{\partial I_e}\right|_{m_d,\omega} = 0. \tag{4}$$

Thus, from (1), (2), and (3a) the condition for minimum losses is given by the following equation:

$$(R_a + C_3\omega^2)i_a^2 - \left(R_e + \frac{\alpha^2(C_1\omega^2 + C_2\omega)}{(1+\beta i_e)^3}\right)i_e^2 = 0. \tag{5}$$

This condition may be simplified if Equation (3b) is used for the dc drive magnetization curve, i.e.:

$$(R_a + C_3\omega^3)i_a^2 - \big(R_e + \alpha^2(C_1\omega^2 + C_2\omega)\big)i_e^2 = 0 \tag{5a}$$

and by substituting i_e from Equation (3b), the minimum losses condition becomes,

$$(R_a + C_3\omega^2)i_a^2 - \left(\frac{R_e}{a^2} + (C_1\omega^2 + C_2\omega)\right)\varphi_e^2 = 0. \tag{5b}$$

Equation (5a) or (5b) shows that the ratio of the armature current to the field current is constant at a given speed regardless of the motor load torque. Thus the motor losses are minimized when the losses due to the armature current become equal to the losses due to the field current. Equation (5b) can be used without loss of generality, given that the dc motor drive losses minimisation causes an inherent reduction of the armature current and thus forces the drive to operate in the linear part of its magnetization curve.

The ultimate objective of this paper is to propose a control strategy that minimizes the power consumption of the robot. In this way, a reduction in economic and environmental cost is achieved. Power consumption is related to the electromechanical losses through,

$$W = \int_0^T \frac{\partial P_{\text{loss}}}{\partial t}\,\mathrm{d}t, \tag{5c}$$

where P_{loss} is given by (1). P_{loss} depends on the excitation strategy of the motor actuator. The improvement in power consumption can thus be measured by calculating (5c) with and without the proposed optimal strategy, yielding W_{opt} and W_n. Their ratio is equivalent to the efficiency ratio,

$$\frac{W_{\text{opt}}}{W_n} = \frac{n_{\text{opt}}}{n},$$

where n_{opt} is the optimal efficiency (i.e. with energy minimization) and n the nominal efficiency (i.e. without energy minimization).

3. Dynamic Model of Robot dc Motor Drives

The differential equations describing the dynamic operation of a robot dc motor drive normalised by the reference nominal values are as follows ([4, 23]):

$$T_a \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{i_a}{i_{0a}}\right) = \frac{u_a}{u_{0a}} - \frac{i_a}{i_{0a}} - \frac{\omega}{\omega_0}\frac{\varphi_e}{\varphi_{0e}}, \quad T_a = \frac{L_a}{R_a}, \tag{6}$$

$$T_{0e} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\varphi_e}{\varphi_{0e}}\right) = \frac{u_e}{u_{0e}} - f\left(\frac{\varphi_e}{\varphi_{0e}}\right), \quad T_{0e} = \frac{\mathrm{N}_e \varphi_{0\varepsilon}}{u_{0e}}, \tag{7}$$

$$T_{mn} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\omega}{\omega_0}\right) = \frac{i_e}{i_{0e}}\frac{\varphi_e}{\varphi_{e0}} - \frac{m_L}{m_{0L}}, \quad T_{mn} = \frac{J\omega_0}{m_{0L}}, \tag{8}$$

$$T_\theta \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\theta}{\theta_0}\right) = \frac{\omega}{\omega_0}, \quad T_\theta = \frac{\theta_0}{\omega_0}, \tag{9}$$

where

$$\frac{\varphi_e}{\varphi_{0e}} = \frac{AI_e}{1 + BI_e} = \Phi_{\text{flux}}, \quad A = \frac{\alpha i_{0e}}{\varphi_{0e}}, \ B = \beta i_{0e}, \ I_e = i_e/i_{0e}$$

and the term $f(\varphi_e/\varphi_{0e}) = i_e/i_{0e}$ is the normalized inverse magnetisation curve.

Equations (6) to (9) are valid under the following assumptions:

- *A*rmature reaction, iron losses and skin effect are negligible.
- The ohmic and induced resistance are temperature indented.
- The voltages u_a, u_e are assumed to be independently controllable.

The dimensionless load torque equation is,

$$\frac{m_L}{m_{0L}} = \frac{m_{L1}}{m_{0L}} + k_L\left(\frac{\omega}{\omega_0}\right), \tag{10}$$

where m_L is the applied load torque, m_{L1} is a disturbance term and k_L is the viscosity coefficient.

The upper and lower limits of electric machine in motoring mode, of the dimensionless normalised state variables, are:

$$0 < I_e < 1 \quad \text{or} \quad 0 < \varphi_e < \frac{A}{1+B}, \quad 0 < V_e < 1, \quad 0 < V_a < 1,$$
$$0 < \Omega < 3, \quad 0 < I_a < 0.2. \tag{11}$$

The limits of 0 and 1 are due to normalisation, the rest are explained in detail in [23].

If the dc motor drive state vector $\boldsymbol{x}(t)$ is defined as,

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} I_a(t) \\ \Omega(t) \\ \Phi_{\text{flux}}(t) \end{bmatrix}$$

and the control vector $\boldsymbol{u}(t)$ is defined as,

$$\boldsymbol{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} V_a(t) \\ V_e(t) \end{bmatrix}$$

then Equations (6)–(9) can be rewritten in state-space form as,

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} -k_1x_1(t) - k_1x_2(t)x_3(t) + k_1u_1(t) \\ k_2x_1(t)x_3(t) - k_2k_3x_2(t) - k_2k_4 \\ \dfrac{-k_5x_3(t)}{k_6 - k_7x_3(t)} + k_5u_2(t) \end{bmatrix} = \boldsymbol{F}\big(\boldsymbol{x}(t), \boldsymbol{u}(t)\big) \tag{12}$$

with boundary conditions,

$$\boldsymbol{x}(0) = [0\ 0\ 0]^{\mathrm{T}} = \boldsymbol{x}_0,$$
$$\boldsymbol{x}(T) = [\text{free}\ 1\ 1]^{\mathrm{T}} = \boldsymbol{x}_T.$$

Equation (12) can better be written in the linear w.r.t. the control variables form,

$$\boldsymbol{F}\big(\boldsymbol{x}(t), \boldsymbol{u}(t)\big) = \boldsymbol{h}\big(\boldsymbol{x}(t)\big) + \boldsymbol{G}\boldsymbol{u}(t), \tag{13}$$

where

$$\boldsymbol{h}\big(\boldsymbol{x}(t)\big) = \begin{bmatrix} -k_1x_1(t) - k_1x_2(t)x_3(t) \\ k_2x_1(t)x_3(t) - k_2k_3x_2(t) - k_2k_4 \\ -\dfrac{k_5x_3(t)}{k_6 - k_7x_3(t)} \end{bmatrix}, \qquad \boldsymbol{G} = \begin{bmatrix} k_1 & 0 \\ 0 & 0 \\ 0 & k_5 \end{bmatrix}. \tag{14}$$

Furthermore, Equation (11) in its state-space form, is,

$$0 < x_1 < 0.2, \qquad 0 < u_1 < 1,$$
$$0 < x_2 < 3, \qquad 0 < u_2 < 1, \tag{15}$$
$$0 < x_3 < \frac{k_6}{1+k_7},$$

where k_6, k_7 are defined in Table I of the Appendix.

4. Optimal Control of Robot dc Motor Drives with Minimization of the Electromechanical Losses

4.1. FIXED SPEED PROFILE

In this case the optimal control problem is to determine the control signals that will excite the dc motor drive so as to follow a desired robot joint speed profile, to satisfy its physical constraints and at the same time to minimize its electromechanical losses.

For the speed control purposes the performance measure is $(x_2(t) - x_{\text{ref}})^2$ where x_{ref} is the required speed. For the minimum control effort the general form of the performance measure is $\boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{u}(t)$, where $\boldsymbol{R}$ is a 2×2 positive definite matrix ([19]).

The total cost function for the whole operation period of the dc motor drive can be defined as:

$$J_m = \int_0^T g\big(\boldsymbol{x}(t), \boldsymbol{u}(t)\big)\, \mathrm{d}t, \tag{16}$$

where $g(\boldsymbol{x}(t), \boldsymbol{u}(t))$ is the sum of the following terms:

- The minimization of the electromechanical losses of the dc motor drive (i.e. Equation (5) or (5a) squared).
- The performance measure for the speed control.
- The performance measure for the minimum effort control.

It is noted that the above cost function minimization achieves total losses minimization of the robot dc motor drive over an operation period of duration T (displacement time).

Furthermore, function $g(\cdot)$ in Equation (16) can be written either as,

$$\begin{aligned} &g_\alpha\big(\boldsymbol{x}(t), \boldsymbol{u}(t)\big) \\ &\quad = p\Big[\big(k_{14} + k_{13}x_2^2(t)\big)x_1^2(t) - \Big(\frac{k_{10}}{k_{11}^2} + k_8x_2^2(t) + k_9x_2(t)\Big)x_3^2(t)\Big]^2 + \\ &\qquad + q(x_2(t) - x_{\text{ref}})^2 + \boldsymbol{u}(t)^{\mathrm{T}}\boldsymbol{R}\boldsymbol{u}(t) \end{aligned} \tag{17}$$

or when the electromechanical losses are not taken into account, as,

$$g_\beta\big(\boldsymbol{x}(t), \boldsymbol{u}(t)\big) = q(x_2(t) - x_{\text{ref}})^2 + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{u}(t), \tag{18}$$

where p and q are weighting factors used to trade off the importance of the electromechanical losses and speed following, and the k_i's are defined in Table I of the Appendix.

The optimal control problem of the robot dc motor drive with simultaneous losses minimization can thus be formulated as follows:

$$\text{minimize} \quad J_m \equiv \int_0^T g\big(\boldsymbol{x}(t), \boldsymbol{u}(t)\big)\,\mathrm{d}t, \tag{19}$$

$$\begin{aligned} \text{subject to} \quad & \dot{\boldsymbol{x}}(t) = \boldsymbol{F}(\boldsymbol{x}(t), \boldsymbol{u}(t)); \quad \boldsymbol{x}(0) = \boldsymbol{x}_0, \boldsymbol{x}(T) = \boldsymbol{x}_T, \\ & \boldsymbol{x}_l \leqslant \boldsymbol{x}(t) \leqslant \boldsymbol{x}_u; \qquad \boldsymbol{u}_l \leqslant \boldsymbol{u}(t) \leqslant \boldsymbol{u}_u, \end{aligned} \tag{20}$$

where

$$\boldsymbol{x}_l = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \qquad \boldsymbol{x}_u = \begin{bmatrix} 0.2 \\ 3 \\ 1 \end{bmatrix}; \qquad \boldsymbol{u}_l = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \qquad \boldsymbol{u}_u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and Equations (20) are the dc motor drive state equations and the operational constraints are given by Equations (6)–(11).

The above dynamic optimisation problem can be transformed in a static nonlinear optimisation problem as follows [19]:

$$\begin{aligned} \text{minimize} \quad f_D &\equiv \Delta t \sum_{k=0}^{K-1} g(\boldsymbol{x}_k, \boldsymbol{u}_k) \\ &= \Delta t \sum_{k=0}^{K-1} g\left(\left(\boldsymbol{x}_k^H + \sum_{l=0}^{k} D_l^k \boldsymbol{u}_l^{i+1}\right), \boldsymbol{u}_k^{i+1}\right) \end{aligned} \tag{21}$$

$$\begin{aligned} \text{subject to} \quad & \boldsymbol{x}_l \leqslant \sum_{j=0}^{K-1} \boldsymbol{D}_j^k \boldsymbol{u}_j^{i+1} + \boldsymbol{x}_k^H \leqslant \boldsymbol{x}_u, \\ & \boldsymbol{u}_l \leqslant \boldsymbol{u}_k^{i+1} \leqslant \boldsymbol{u}_u, \end{aligned} \tag{22}$$

where

$$\boldsymbol{x}_{k+1}^H = \boldsymbol{A}_k^i \boldsymbol{x}_k^H + \boldsymbol{c}_k^i; \qquad \boldsymbol{x}_1^H = \boldsymbol{A}_0 \boldsymbol{x}_0 + \boldsymbol{c}_0; \qquad \boldsymbol{x}_0^H = \boldsymbol{x}_0 \tag{23}$$

$$\boldsymbol{D}_j^{k+1} = \begin{cases} \boldsymbol{A}_k^i \boldsymbol{D}_j^k & \text{for } k > j, \\ \boldsymbol{B}_k^i & \text{for } k = j, \\ \boldsymbol{0} & \text{for } k < j \end{cases} \tag{24}$$

and $K \Delta t = T$.

The term $\boldsymbol{x}_{k+1}^H$ is the part of the solution for $\boldsymbol{x}_{k+1}^{i+1}$ that does not depend on the control values $\boldsymbol{u}_0^{i+1}, \boldsymbol{u}_1^{i+1}, \ldots, \boldsymbol{u}_{k-1}^{i+1}$, and $\boldsymbol{D}_j^{k+1}$ is an $n \times m$ matrix that determines the contribution of the control at the jth instant to the state value at the $(k+1)$st instant. The matrix $\boldsymbol{D}$ will contain an upper triangular matrix of zeros, and the matrices $\boldsymbol{D}_j^0$, $j = 1, \ldots, K-1$, in the top row are all defined to be zero. (Note that the superscript $k+1$ on the matrix $\boldsymbol{D}$ does not indicate the $(k+1)$st power of $\boldsymbol{D}$.) In general,

$$\boldsymbol{x}_{k+1}^{i+1} = \boldsymbol{x}_{k+1}^{H} + \sum_{j=0}^{k} \boldsymbol{D}_j^{k+1} \boldsymbol{u}_j^{i+1}, \tag{25}$$

$$\boldsymbol{x}_{k+1}^{i+1} = \boldsymbol{A}(\boldsymbol{x}_k^i, \boldsymbol{u}_k^i) + \boldsymbol{B}(\boldsymbol{x}_k^i, \boldsymbol{u}_k^i)\boldsymbol{u}_k^{i+1} + \boldsymbol{c}(\boldsymbol{x}_k^i), \tag{26}$$

$$\boldsymbol{A}_k^i = \boldsymbol{A}(\boldsymbol{x}_k^i, \boldsymbol{u}_k^i) = \frac{\partial \boldsymbol{x}_{k+1}(\boldsymbol{x}_k^i, \boldsymbol{u}_k^i)}{\partial \boldsymbol{x}}, \tag{27}$$

$$\boldsymbol{B}_k^i = \boldsymbol{A}(\boldsymbol{x}_k^i, \boldsymbol{u}_k^i) = \frac{\partial \boldsymbol{x}_{k+1}(\boldsymbol{x}_k^i, \boldsymbol{u}_k^i)}{\partial \boldsymbol{u}}, \tag{28}$$

$$\boldsymbol{c}_k^i = \boldsymbol{c}(\boldsymbol{x}_k^i, \boldsymbol{u}_k^i) = \boldsymbol{x}_{k+1}(\boldsymbol{x}_k^i, \boldsymbol{u}_k^i) - \frac{\partial \boldsymbol{x}_{k+1}(\boldsymbol{x}_k^i, \boldsymbol{u}_k^i)}{\partial \boldsymbol{x}}\boldsymbol{x}_k^i - \frac{\partial \boldsymbol{x}_{k+1}(\boldsymbol{x}_k^i, \boldsymbol{u}_k^i)}{\partial \boldsymbol{u}}\boldsymbol{u}_k^i. \tag{29}$$

From Equations (14) and (26)–(29) matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and vector $\boldsymbol{c}$ are given by the relations,

$$\boldsymbol{A}_k^i = \begin{bmatrix} 1-\Delta t k_1 & -\Delta t k_1 x_k^i(3) & -\Delta t k_1 x_k^i(2) \\ \Delta t k_2 x_k^i(3) & 1-\Delta t k_2 k_3 & \Delta t k_2 x_k^i(1) \\ 0 & 0 & 1-\dfrac{\Delta t k_5 k_6}{(k_6 - k_7 x_k^i(3))^2} \end{bmatrix}, \tag{30}$$

$$\boldsymbol{B}_k^i = \begin{bmatrix} \Delta t k_1 & 0 \\ 0 & 0 \\ 0 & \Delta t k_5 \end{bmatrix}, \tag{31}$$

$$\boldsymbol{c}_k^i = \begin{bmatrix} \Delta t k_1 x_k^i(2) x_k^i(3) \\ \Delta t k_2 x_k^i(1) x_k^i(3) - \Delta t k_2 k_4 \\ \dfrac{\Delta t k_2 k_4 (x_k^i(3))^2}{(k_6 - k_7 x_k^i(3))^2} \end{bmatrix}. \tag{32}$$

Function $g(\cdot)$ in the integral of the cost function, Equation (19), can be Equation (17), $g_\alpha(\cdot)$, when the electromechanical losses are taken into account or Equation (18), $g_\beta(\cdot)$, when they are not.

4.2. OPTIMAL SPEED PROFILE

The determination of the optimal speed trajectory for a direct or indirect driven robot with simultaneous minimization of its electromechanical losses is a nonlinear constrained optimisation problem. Under this consideration, the robot arm rotates with a given load through a definite angle in limited time while the dc motor actuator is required to consume the minimum possible input power.

The power losses in a dc motor actuator are given by Equation (1). The energy losses over an operation cycle of an incremental motion dc drive are given by,

$$W = \int_0^T g(i_e, \omega, p_i)\,\mathrm{d}t, \tag{33}$$

where T is the displacement duration time, $g(i_e, \omega, p_i)$ is the function of the controllable losses given by Equation (17) or (18), and p_i denotes the motor parameters i.e. k_L, the viscous friction of the load, m_{0L} the extrapolated stalled rotor torque and J the total mass and load inertia of the motor.

The above optimization problem is formulated as following:

minimize W (i.e. Equation (33)) (34)

subject to: *dc motor differential equations* (Equations (6) to (9)),
operational limits of dc motor drive (Equation (15)), and
angular velocity boundary conditions: $\omega(0) = \omega(T) = 0$.

The displacement duration time T, must be compared with the mechanical time constant of the load T_{mn}.

5. The Numerical Algorithm

In order to solve the static non-linear constrained optimisation problem, the following 5-step algorithm is implemented:

1. Using the state-control history $\boldsymbol{x}^i$, $\boldsymbol{u}^i$, calculate matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and vector $\boldsymbol{c}$, and use these quantities to determine $\boldsymbol{X}^H$ and $\boldsymbol{D}$.
2. Substitute the numerical values of $\boldsymbol{X}^H$ and $\boldsymbol{D}$ into the expressions in Equation (22) to determine the coefficients in the constraining equations and the performance measure.
3. Minimize f_D, using the Sequential Quadratic Programming (SQP) algorithm [2].
4. Determine $\boldsymbol{x}^{i+1}$ by evaluating the following Equation (35) using $\boldsymbol{u}^{i+1}$ obtained in step 3

$$\boldsymbol{x}^{i+1} = \boldsymbol{D}\boldsymbol{u}^{i+1} + \boldsymbol{X}^H. \tag{35}$$

5. If the norm of the difference between successive control iterates is small, that is,

$$\|\boldsymbol{u}^{i+1} - \boldsymbol{u}^i\| \leqslant \gamma \tag{36}$$

terminate the procedure and output $\boldsymbol{x}^{i+1}$, $\boldsymbol{u}^{i+1}$ and the minimum value of f_D; otherwise increase i by one and return to step 1.

A digital computer program was realized in order to perform steps 1 through 5 using Matlab® [28]. The Sequential Quadratic Programming (SQP) method is performed. An estimate of the Hessian and Lagrangian of the optimization problem of Equations (21) and (22) is updated at each iteration, using the BFGS (Broyden–Fletcher–Goldfarb–Shanno) formula [28]. A line search is performed using a merit function similar to that proposed by Han [14] and Powell [30, 31]. The QP subproblem is solved using an active set strategy similar to that described in Gill, Murray, and Wright [13]. The function to be minimized and the constraints must both be continuous functions w.r.t. their variables. When the problem is infeasible, the algorithm attempts to minimize the maximum constraint value.

Moreover, a first order approximation of the differential equation (19) is very sensitive to numerical errors. In order to improve the quality of the approximation involving only a few steps in the time interval, higher order approximation and a predictor-corrector-method is used [2]. Expanding the state trajectory into a Taylor series would force to compute the higher derivatives. For this purpose the Runge–Kutta formula is used [28].

6. Simulation Studies

A number of simulations are performed in order to evaluate the performance of the proposed algorithm. In the simulation three motor sizes were considered:

1. 1 kW (rated value: rotational speed 2000 rpm, armature voltage 400 V, armature current 2.85 A, 2 poles).
2. 3 kW (rated value: rotational speed 2000 rpm, armature voltage 400 V, armature current 8.2 A, 4 poles).
3. 60 kW (rated value: rotational speed 2000 rpm, armature voltage 500 V, armature current 120 A, 4 poles).

The first 2 sizes are used in common robots applications [5]. The third one is industrial and is used to show that the savings achieved with the proposed algorithm increase with the size of the motor and that the proposed methodology is equally useful to non-robotic applications.

The proposed control scheme for the determination of the optimal excitation of a robot dc motor actuator with simultaneous minimisation of its electromechanical losses, by following a desired speed profile under a fluctuating load is named "Test1". To demonstrate the merits of the proposed control scheme the results of "Test1" are compared with the results of another procedure named "Test 2", which controls the speed of the robot dc motor drive using minimum effort control, without simultaneous minimization of its electric losses. Both control schemes are executed for various motor loads.

The simulation experiments are carried out for an operation period of 1s. The characteristics for the above tested dc motors are given in the Appendix. The values of the weighting factors $p, q, \boldsymbol{R}$ in the cost function Equations (17) and (18), are set as follows:

$$p = 20, \qquad q = 10, \qquad \boldsymbol{R} = \begin{bmatrix} 20 & 10 \\ 10 & 20 \end{bmatrix}.$$

All the k_i's (k_1 to k_{12}, except k_4), which are involved in the relevant equations are calculated from the motor characteristics, see Table I of the Appendix.

To find a global minimum, several starting points are tried and the motor load, k_4, is varied with a Gaussian random disturbance $\pm 20\%$, in order to determine

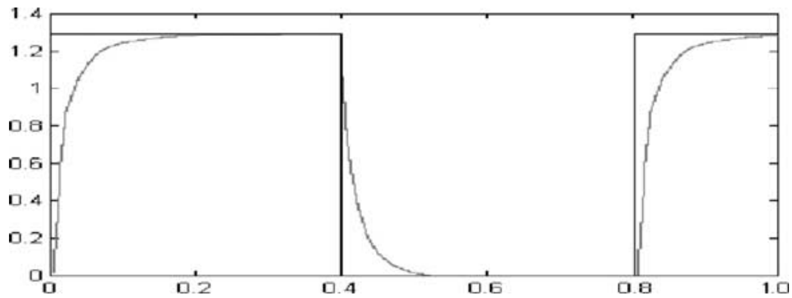

Figure 1. Robot dc motor speed responce as derived by the proposed control scheme "Test 1" (Speed control with simultaneous electromechanical losses minimization).

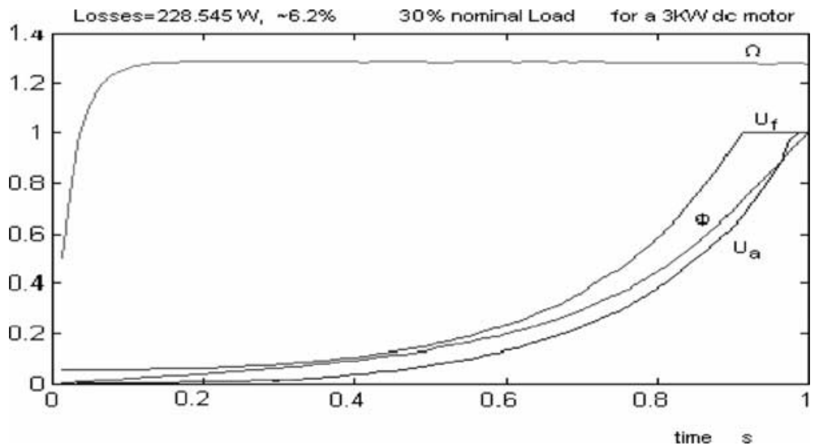

Figure 2a. The optimal excitation of a robot dc motor actuator as derived by the proposed control scheme "Test1" (Minimal effort control, with Electromechanical losses minimization, with simultaneous Speed control, while it is following a desired speed profile under a 30% nominal load).

as many local minima as possible. To compare the implemented speed control approaches the reference speed was $\Omega = 1300$ rpm, generating a rectangular waveform. For the three dc motor drives used the speed response (Figure 1) showed no overshoot and it has approximated closely the reference value.

Figure 2(a) shows the optimal excitation of a 3 kW dc motor actuator with minimal control effort and simultaneous minimisation of its electromechanical losses while it is following a desired speed profile of 1300 rpm (control scheme "Test1"), under a 30% nominal load. Figure 2(b) shows the optimal excitation of the same 3 kW dc drive without minimization of its electromechanical losses, which is the control scheme "Test2". In Figures 2(a), 2(b) it is noticed that the speed response excitation (Ω) tends slowly to its optimum after steady state has been reached. An energy saving of 6.2%, compared with "Test2", is observed for the whole operation period. Moreover, smooth excitation signals without fluctuations and peaks are provided, which are very important in practical implementations. Figures 3, 4 and

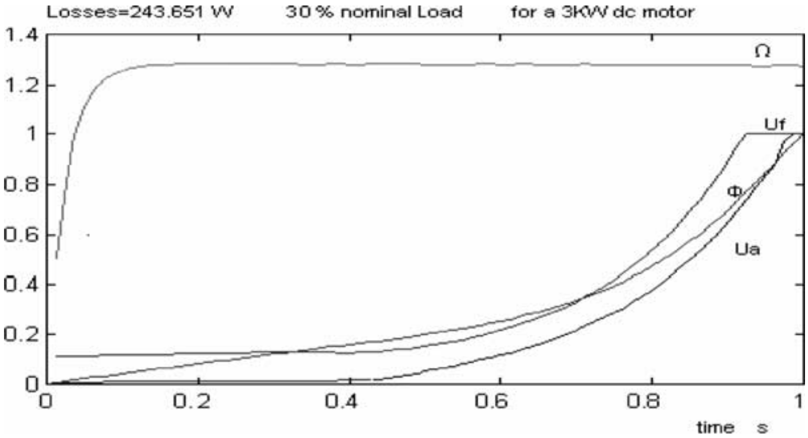

Figure 2b. The optimal excitation of a robot dc motor actuator as derived by the control scheme "Test2" (Minimal effort control, without minimization of Electromechanical losses, with simultaneous Speed control, while it is following a desired speed profile under a 30% nominal load).

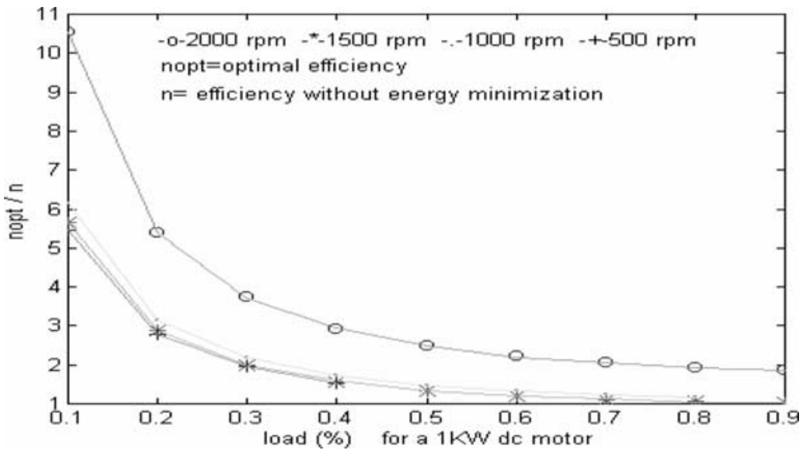

Figure 3. Experimental study for a 1 kW robot dc motor drive: 2%–8% efficiency improvement by the proposed control scheme "Test1" compared with the control scheme "Test2".

5 show the ratio of the optimal efficiency that is achieved by the proposed control scheme "Test1" compared with the control scheme "Test2", for the 1 kW, 3 kW and 60 kW drives respectively. The resulted improvement is up to 2% for heavy loads and 8% for light loads.

Determination of the Optimal Speed Trajectory

For this simulation an 1 kW robot dc motor drive is used. The optimization problem, (34), is solved using the proposed control scheme "Test1", described previously. Using different displacement times T, it was found that the optimum speed

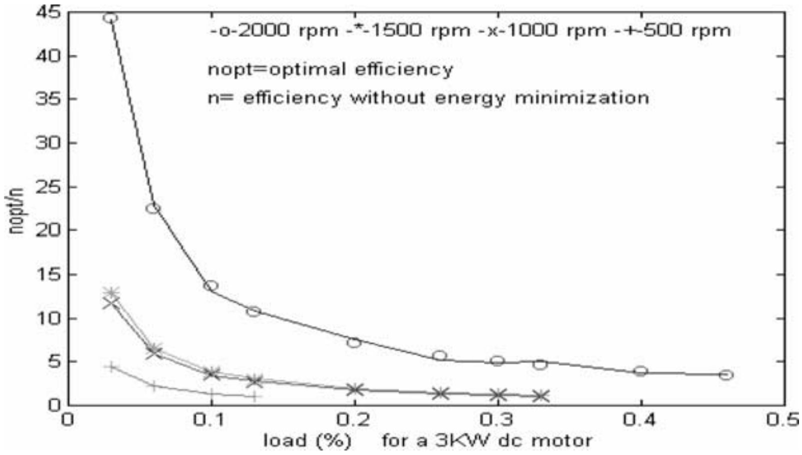

Figure 4. Experimental study for a 3 kW robot dc motor drive: 2%–8% efficiency improvement by the proposed control scheme "Test1" compared with the control scheme "Test2".

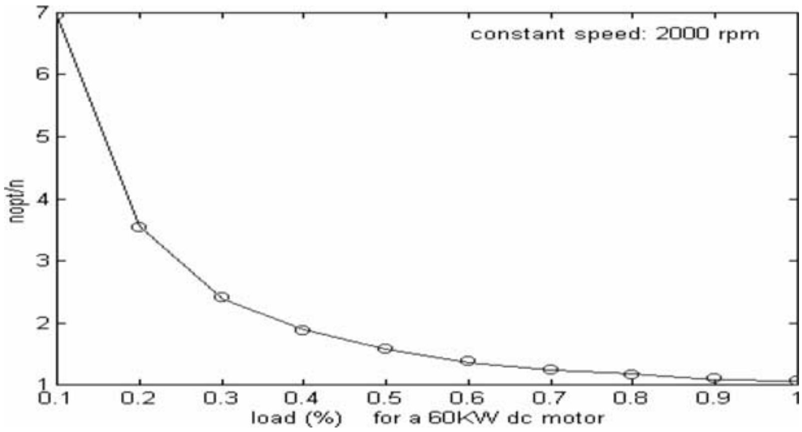

Figure 5. Experimental study for a 60 kW robot dc motor drive: 2%–8% efficiency improvement by the proposed control scheme "Test1" compared with the control scheme "Test2".

profile depended on $k = T_{mn}/T$. The optimal speed profiles are plotted in Figures 6 and 7, as speed per unit versus time per unit, where the per unit speed is taken as the ratio ω/ω_{0m} and the per unit time as the ratio $\tau = t/T$. What these plots show is that if $k > 0.1$ the speed profiles are parabolic instead of the widely used trapezoidal, while for values of $k < 0.01$ our experiments suggest the trapezoidal profile.

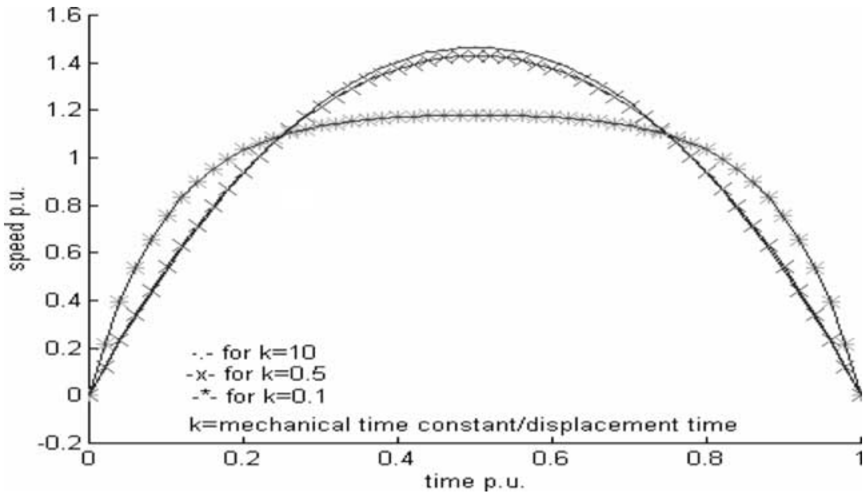

Figure 6. Optimal speed trajectories of incremental motion robot dc drives for high ($\geqslant 0.1$) mechanical time constant.

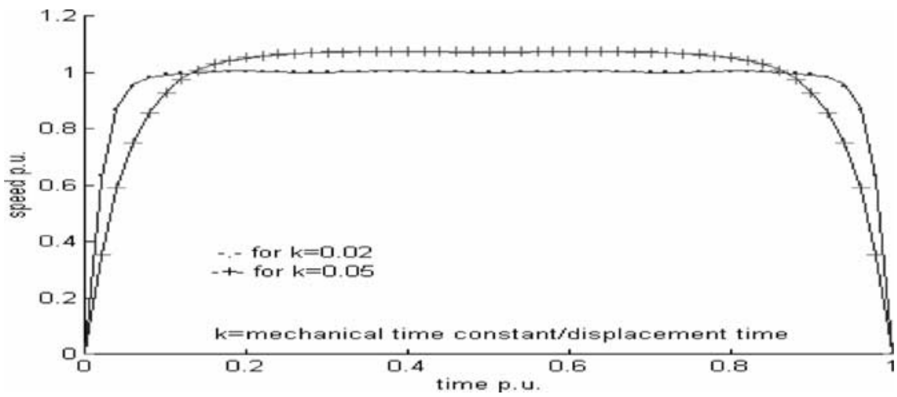

Figure 7. Optimal speed trajectories of incremental motion robot dc drives for low (< 0.01) mechanical time constant.

7. Conclusions

An innovative method for optimally efficient dc motor drive speed regulation is proposed. This method is derived using concepts of optimal control theory and is implemented easily in the Matlab® environment. The control signals derived by the proposed control scheme can be implemented through lookup table techniques. The main advantages of the proposed control scheme are:

- The proposed method does not alter the motor drive dynamics.
- It is easy in implementation.
- The same method may be applied to different categories of motor actuators.

The speed trajectory is shown to depend on the motor mechanical constant/displacement time and does not necessarily follow the widely used trapezoidal profile. As a result of the optimum speed profile utilization, significant energy savings of the order of 3%–8% can be achieved in industrial robot dc motor drives. This in turn results in less operational cost but also in an overall environmental profit due to the decrease in the amount of required energy.

Appendix

Table I. dc motor constants (see [22])

Parameters		Nominal prices		
		1 kW motor 2000 rpm	3 kW motor 2000 rpm	60 kW motor 2000 rpm
ω_0	no-load speed (base speed) (rad /s)	209.2	209.2	209.2
u_{0a}	armature voltage ($c\varphi_{0\varepsilon}\omega_0$), (V)	400	400	500
i_{0a}	extrapolated stalled rotor current ($u_{\alpha 0}/R_\alpha$) (A)	29.85	62.5	62.5
i_a	rotor current (A)	2.85	8.2	120
u_{0e}	stator (field) voltage ($R_e i_{0\varepsilon}$), (V)	180	180	180
i_{0e}	stator (field) current (A)	0.7	1.1	4.3
m_{0L}	extrapolated stalled rotor torque (Nm)	55.7	117.5	271.7
R_α	rotor resistance (Ω)	13.4	6.4	0.217
L_α	rotor inductance (H)	0.09	0.03	0.03
R_e	field resistance (Ω)	257	163	116
L_e	field inductance (H)	42.11	70	70
c	constant (V s rad^{-1}) or (NmA^{-1})	1.84	1.88	1.88
J_m	motor mass inertia (kg m^2)	$6.199 \cdot 10^{-3}$	$1.711 \cdot 10^{-3}$	$1.711 \cdot 10^{-3}$
J	motor mass and load inertia (kg m^2)	$28.46 \cdot 10^{-3}$	$28.46 \cdot 10^{-3}$	$28.46 \cdot 10^{-3}$
m_{L1}	disturbance term	$\pm 0.2 m_{0L}$	$\pm 0.2 m_{0L}$	$\pm 0.2 m_{0L}$
P	poles of the machine	2	4	4
m	weight of the steel core	4.959	8.760	8.76
M	the total mass of rotor	3.732	6.648	6.648
k	the percentage of steel in the gross core	0.75	0.75	0.75
ρ	density of the steel (Kg/m^3)	7600	7600	7600
l	length of the core [mm]	80	90.5	90.5
λ	ratio of stator contact surface to the rotor surface	2/3 to 3/4	2/3 to 3/4	2/3 to 3/4
N_e	winding (turns)	5740	1350	1350
K_s	saturation factor	1.8	1.8	1.8
C_3	coefficient of stray losses	0.15	0.15	1.7

Table I. (Continued)

a	constant corresponding to the linear part of the magnetization curve	0.0073	0.052	0.052
β	constant corresponding to the non linear part of the magnetization curve	2.286	0.421	0.421

$$k_1 = 1/T_a = R_a/L_a, \qquad k_8 = C_1 \quad \text{(see Table III)},$$
$$k_2 = 1/T_{mn} = m_0/J\omega_0, \qquad k_9 = C_2 \quad \text{(see Table III)},$$
$$k_3 = K_L = k_L\omega_0/m_{0L}, \qquad k_{10} = R_e,$$
$$k_4 = m_{L1}/m_{0L}, \qquad k_{11} = \alpha = L_e/N_e,$$
$$k_5 = 1/T_{0e} = u_{0\alpha}/N_e\Phi_{0e} = R_e/L_e, \qquad k_{12} = i_{0a}/i_{0e} = \beta,$$
$$k_6 = A = \alpha i_{0e}/\varphi_{0e} = 1 + \beta i_{0e}, \qquad k_{13} = C_3,$$
$$k_7 = B = \beta i_{0e}, \qquad k_{14} = R_\alpha.$$

Table II. Magnetic losses of iron laminations according to DIN 46400

Type	Losses (W/kg)	Thickness (mm)	Hysteresis losses (%)	Eddy-current losses (%)	σ_H	σ_F
I	3.6	0.5	66.7	33.3	4.8	4.8
I	3.15	0.35	74.6	25.4	4.7	3.2
II	3	0.5	78.3	21.7	4.7	2.6
III	2.3	0.5	82.5	17.5	3.8	1.6
III	2	0.5	73	27	2.92	2.16
IV	1.7	0.5	83.8	16.2	2.85	1.10
IV	1.55	0.3	88	12	2.73	0.74
IV	1.35	0.35	88.9	11.1	2.4	0.60
IV	1.20	0.35	75	25	1.8	1.2

Table III.

Type	Lamination thickness (mm)	C_1	C_2
I	3.6	0.14	119.35
III	2.3	0.11	39.77
III	2	0.08	53.69

C_1 and C_2 are calculated by,

$$C_1 = \frac{P^2}{16 \cdot 10^4 \pi^3 \lambda^2 l} k\rho\sigma_F = 1.532 \frac{P^2}{\lambda^2 l[mm]} k\sigma_F, \quad (37)$$

$$C_2 = \frac{P^2}{800\pi^2 \lambda^2 l} k\rho\sigma_H = 962.551 \frac{P^2}{\lambda^2 l[mm]} k\sigma_H, \quad (38)$$

where

P the poles of the machine,

m the weight of the armature's core, $m = k\rho V$,

ρ is the density of the steel (7600 Kg/m^3),

V the armature volume (cross-sectional area throughout the length of the core x magnetic path length) and the steel occupies k times gross core volume,

l the length of the core [mm],

k the percentage of steel in the gross core,

λ is the rate of stator conduct surface to the rotor's surface. This exists under the assumptions that stator's field is uniform, and the armature has a cylindrical form.

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